Hypotrochoid line animations

Interactive, colorful, and seemingly endless animations created with mathematical functions of hypotrochoids and lines, with a hint of moiré.

Inspiration

Moiré patterns were the first inspiration for this project. I was trying to generate interesting patterns, when this idea popped up to create not two different patterns that would overlap to create a moiré but instead have one figure overlap itself. Then I came across this video of John Whitney, who made some fascinating animations. The idea of generating moiré patterns moved to the background, but they are still visible!

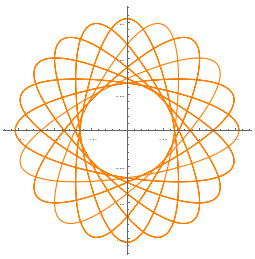


Figure 2: Hypotrochoid 2

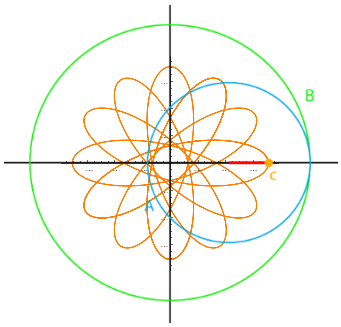


Figure 1: Hypotrochoid creation, hypotrochoid 1

Line animations

These animations are created by a program drawing lines between points of two hypotrochoids. A **hypotrochoid** is a figure created by a point c attached to circle A of radius r which rolls around the inside of a fixed circle B with radius R. The point c is a distance d from the center of A.
The parametric equations for hypotrochoids are:

$$\begin{cases} x = (R-r)\cos(t) + d\cos\left(\frac{R-r}{r}t\right) \\ y = (R-r)\sin(t) - d\sin\left(\frac{R-r}{r}t\right) \end{cases}$$

Interactivity

In the interactive artwork, users can play around with the variables R, r and d of each of the two hypotrochoids using sliders, to see how the animation changes and to create their own animations. The variables have been set to predefined values before the user changes them, which I refer to as "standard values".

The specific instance drawn in figure 1 has values $R_1 = 1.0,\ r_1 = 0.58,\ d_1 = r_1/2$ (which are the standard values of one of the hypotrochoids in the artwork). The other hypotrochoid in the artwork (drawn separately in figure 2) has standard values $R_2 = 2.0,\ r_2 = 0.9,\ d_2 = r_2/2$.

A zoom variable was added, such that the user can zoom-in or -out on the two hypotrochoids, to create more interesting figures. This leads to the following parametric equations:

$$\begin{cases} x = zoom * ((R-r)\cos(t) + d\cos\left(\frac{R-r}{r}t\right)) \\ y = zoom * ((R-r)\sin(t) - d\sin\left(\frac{R-r}{r}t\right)) \end{cases}$$

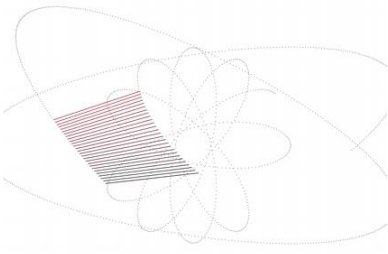
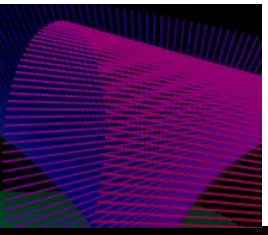


Figure 3: lines

Each frame, t increases by one, and $numberOfLines$ (another variable that can be set by the user) lines are drawn in a loop. These lines start at coordinate $(x_1(timefactor_1 * (t+i)),\ y_1(timefactor_1 * (t+i)))$ and end at $(x_2(timefactor_2 * (t+i)),\ y_2(timefactor_2 * (t+i)))$, where i is an integer and $numberOfLines > i \geq 0$. The factors $timefactor_1$ and $timefactor_2$ are added to control the time (and thus space) between the points in the hypotrochoids, and thus also control the space between the lines between the two hypotrochoids.

Figure 4: a line moiré



Moiré

An interesting phenomenon is visible in certain points in the animation (see figure 4): moiré patterns are visible. A line moiré is an interference pattern where a pattern with lines followed by transparent gaps is displayed on top of a similar but slightly different pattern. It can also occur when the screen resolution is not optimal for the line width. What happens is that at some points the gaps between the overlapping lines are smaller than between other points. This visually suggests new lines or shapes.